M259. *Proposed by the Mayhem Staff.*

The number n is formed by concatenating the strings of digits formed by the numbers 2^{2006} and 5^{2006}. How many digits does n have?

Solution by Arkady Alt, San Jose, CA, USA.

More generally, for any natural number m, let p and q be the number of digits in the strings of digits formed by 2^m and 5^m, respectively. Then $10^{p-1} < 2^m < 10^p$ and $10^{q-1} < 5^m < 10^q$. Therefore,

$$\left(10^{p-1}\right)\left(10^{q-1}\right) \;<\; 2^m \cdot 5^m \;<\; 10^p \cdot 10^q\,;$$

that is,

$$10^{p+q-2} \;<\; 10^m \;<\; 10^{p+q}\,.$$

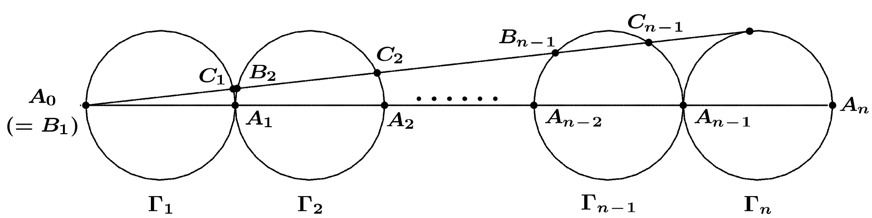
Thus, $p+q-2 < m < p+q$, which is equivalent to $m = p+q-1$, or $p+q = m+1$. We can conclude that a concatenation of 2^m and 5^m has $m+1$ digits. In particular, taking $m = 2006$, we find that n has 2007 digits.

Also solved by HOUDA ANOUN, Bordeaux, France; ALPER CAY, Uzman Private School, Kayseri, Turkey; HASAN DENKER, Istanbul, Turkey; RICHARD I. HESS, Rancho Palos Verdes, CA, USA; JEAN-DAVID HOULE, student, McGill University, Montreal, QC; D. KIPP JOHNSON, Beaverton, OR, USA; and KUNAL SINGH, student, Kendriya Vidyalaya School, Shillong, India.

M260. *Proposed by Bruce Shawyer, Memorial University of Newfoundland, St. John's, NL.*

Points A_0, A_1, …, A_n lie on a line, in that order, spaced a uniform distance $2r$ apart. For $1 \le k \le n$, let Γ_k be the circle with $A_{k-1}A_k$ as diameter. The line through A_0 tangent to Γ_n intersects the circle Γ_k at the points B_k and C_k, for $1 \le k \le n-1$.

Determine the length of the line segment B_kC_k for $1 \le k \le n-1$.



Solution by Richard I. Hess, Rancho Palos Verdes, CA, USA.

Let O_k be the centre of Γ_k. Let M_k be the mid-point of chord B_kC_k for $1 \le k \le n-1$, and let M_n be the point of tangency to Γ_n.